R&D in the Network of International Trade: Multilateral versus Regional Trade Agreements *

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November 10, 2008

Abstract

Recent empirical evidence has shown that trade liberalization promotes productivity growth in individual firms. This paper argues that different types of trade liberalization – multilateral versus regional – may lead to different productivity at the firm’s level. Trade agreements between countries are modelled with a network: nodes represent countries and a link indicates a trade agreement between the linked countries. In this framework, the multilateral trade agreement is represented by the complete network while the overlap of regional trade agreements is represented by the hub-and-spoke trade system. Trade liberalization, which increases the network of trade agreements, affects incentives for firms to invest in costly productivity-enhancing activity, such as research and development (R&D), via two major mechanisms. It reinforces the incentives for R&D through the creation of new markets (scale effect) but it may also dampen these incentives through the emergence of new competitors (competition effect). The joint action of these two effects within the multilateral and the regional trade systems gives rise to the result that, for the same number of direct trade partners, the R&D effort of a country in the multilateral agreement is lower than the R&D effort of a hub but higher than the R&D effort of a spoke. This suggests that productivity gains of regionalism versus those of multilateralism depend heavily on the number of regional trade agreements signed by a country. If a country signs sufficiently many regional trade agreements (strong negotiator), then its R&D and productivity are higher in the regional trade system than in the multilateral system. At the same time, if a country signs only a few trade agreements (weak negotiator), its productivity gains in the regional trade system are lower.

JEL Classification: O31, D85, D43, F13

Keywords: Trade, multilateralism, regionalism, R&D, network, oligopolistic competition

*I would like to thank my advisors Morten Ravn and Fernando Vega-Redondo for guidance, helpful comments and support. I also thank the research staff of the WTO, especially Michele Ruta, Patrick Low and Roberta Piermartini, the participants of the 6th ELSNIT conference in Florence on October 24-25, and Francis Bloch for useful suggestions and positive feedback.

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1 Introduction

In the era of unprecedented proliferation of regional trade agreements and simultaneous developments in the WTO, assessments of relative economic benefits of multilateralism versus regionalism takes on special significance. Numerous studies investigate the difference in welfare benefits, trade volumes, GDP levels and GDP growth rates across multilateral and regional trade arrangements.\(^1\) However, the existing literature has not examined the issue of possible variations in the impact of different types of trade liberalization on countries’ \textit{productivity}.

The latter is surprising for at least two reasons. First, productivity is a key determinant of aggregate output, which is found to vary across different types of trade arrangements. Secondly, recent empirical evidence has shown that in general trade liberalization has a \textit{significant effect} on the productivity level of the country. Bustos (2007) finds that during the period of trade liberalization between Argentina and Brazil, companies in sectors benefiting from a comparatively higher reduction in Brazil’s tariffs increased their spending on purchases of technology goods. Likewise, Trefler (2004) observes that the U. S. tariff concessions caused a boost in labor productivity of the Canadian firms in the most impacted, export-oriented group of industries. Similar patterns are shown by Bernard et al. (2006) for the U. S., by Topolova (2004) for India, by Aw et al. (2000) for Korea and Taiwan, by Alvarez and Lopez (2005) for Chile, and by Van Biesebroeck (2005) for sub-Saharan Africa.\(^2\) Additionally, the positive effect of trade liberalization on productivity is substantiated by the extensive theoretical work.\(^3\)

The aim of this paper is to contribute to the literature on the impact of trade liberalization on firms’ productivity by studying how this impact varies across two types of trade liberalization – multilateral versus regional. I consider a model in which firms can improve their productivity by investing in costly R&D. Here R&D is viewed broadly as any activity aimed at reducing the marginal cost of production.\(^4\)

I study the mechanisms through which trade can affect the return to innovation within different

\(^1\) For an extensive research of theoretical models on this subject see Panagariya (2000). The empirical works are summarized in De la Torre and Kelly (1982), Srinivasan et al. (1993), and Frankel (1997). Other theoretical and empirical works include Krueger (1999), Bhagwati (1993), Kowalczyk and Wonnacott (1992), Deltas et al. (2005), Goyal and Joshi (2006), Diao et al. (2003).


\(^3\) The theoretical models identify several channels through which international trade affects productivity at the industry and/or at the firm level: the improved allocation of resources through specialization (Grossman and Helpman (1991)), the knowledge spillovers effect (Rivera-Batiz and Romer (1991), Devereux and Lapham (1994)), the reallocation of economic activities from less to more productive firms (Melitz (2003), Bernard et al. (2007), Yeaple (2005)), the exploitation of economies of scale (Krugman (1980)), the pro-competitive effect of trade openness (Aghion et al. (2005), Peretto (2003), Licandro and Navas-Ruiz (2007)), and others. See World Trade Report 2008 for the survey.

\(^4\) Examples include developing new production technology, training of employees, internal re-organization of resources and factors of production.
types of trade systems. The two major mechanisms are the *scale* effect due to the increased size of the market and the *competition* effect due to the increased number of competitors in the markets. The focus of this paper is on the interaction of the two effects within multilateral and regional types of trade agreements.

I model trade agreements between countries with a network. Nodes represent countries and a link between the nodes indicates the existence of a trade agreement. In every country, there is a single firm producing one good. The good is sold domestically and in markets of the trade partner countries subject to oligopolistic competition. There is, therefore, an intra-industry trade between countries which are directly linked in the network.

The advantage of modelling trade agreements with a network is that it enables distinction between various types of trade systems. In particular, it allows me to focus on such differences between trade systems as the degree of countries’ trade involvement (the number of trade agreements signed) and the nature of market interaction between countries (who trades and competes with whom, on which markets, how many traders are present in each market, etc). Given the focus of the paper on interaction between the scale and the competition effects of trade liberalization within different types of trade systems, capturing exactly these differences is key.

I constrain the analysis to two specific classes of network structures associated with the multilateral and the regional scenarios of trade liberalization. The first class of networks is symmetric, or regular, networks. It incorporates the case of a *complete* network structure – a network where any one country is directly linked to every other country. The complete network in this model represents the multilateral trade agreement. The second class of networks is asymmetric networks with two types of nodes: high and low degree nodes. This class of networks captures the basic characteristics of the so-called *hub-and-spoke* trade system, where some countries (hubs) have relatively large number of direct trade partners as compared to other countries (spokes), which are mainly involved in trade agreements with hubs. According to a number of contributions on regional trade agreements, the hub-and-spoke trade arrangement has become a typical outcome of the regional trade liberalization.5

The modelling approach in this paper is closely related to the common approach in the strand of literature on R&D *co-operation* between firms in oligopoly. This strand of literature is well represented by the seminal papers D’Aspremont and Jacquemin (1988) and Goyal and Moraga (2002). They consider a framework of Cournot competition, where at a pre-competitive stage firms

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5To my knowledge, the concept of hub-and-spoke trade arrangement was first introduced in Lipsey (1990) and Wonnacott (1990). It was further developed in Lipsey (1991), Wonnacott (1991, 1996), Kowalczyk and Wonnacott (1991, 1992), Baldwin (2003, 2005), De Benedictis et al. (2005), and others.
can exert a cost-reducing effort. The typical element of this approach is that the rationale of co-operation between firms is the existence of R&D spillovers, which creates an externality. Co-operation is intended to internalize such an externality.

Similarly to models with R&D co-operation, in this model, firms compete in a Cournot fashion choosing individual R&D efforts and production levels in a two-stage non-cooperative game. However, in contrast to D’Aspremont and Jacquemin (1988), Goyal and Moraga (2002) and the related literature, I abstract from R&D collaboration and spillovers. Instead, I concentrate purely on the effects of market access and competition faced by firms within various types of trade agreements on innovation intensity of the firms. Besides, unlike the assumption of standard oligopolistic competition between firms in one common market imposed in D’Aspremont and Jacquemin (1988) and Goyal and Moraga (2002), the central assumption in the present framework is that firms compete not only in one, but in several separate markets and every market is accessible only to those firms which have a trade agreement with that market. Clearly, this assumption results in heterogeneity between firms in terms of their market size, a feature which is absent from the previous models.

The primary result of the paper is that the impact of trade liberalization on firms’ R&D efforts depends crucially on the features of the trade agreements. Basically, the sizes of the scale effect and the competition effect, due to a new trade partner, vary across the multilateral and the regional trade systems since both effects are predetermined by the structure of the trade system. For example, with regard to the scale effect, gaining access to a new market in either a multilateral or bilateral context enhances incentives for firms to innovate. Yet, the ”net worth”, or the effective size, of the new market depends on the number of other firms present in the market and on the competitive power of these firms – their R&D and production levels. Those are both determined by the structure of the trade system. Similarly, with respect to the competition effect, a new trade partner of a firm both in the multilateral and in the regional agreement becomes an additional rival of the firm in its domestic market. However, depending on the structure of the trade system, it may also become a rival in some, all or none of the firm’s foreign markets. In addition, the size of the market share obtained by the new rival in firm’s domestic and foreign markets depends on the number and on the competitive strength of other firms present in these markets, as well as on the competitive strength of the rival himself. Those are defined by the structure of the trade system and by the market interactions of the firms with their own trade partners.

The difference in the scale and in the competition effects of trade liberalization and the resulting difference in the effective sizes of markets across the multilateral and the regional trade systems leads to the variation in levels of R&D efforts across systems. I show that for the same number of direct
trade partners, the R&D effort of a hub in the regional trade system is higher than that of a country in the multilateral agreement. On the other hand, the R&D effort of a spoke is lower than that of a hub and lower than the R&D effort of a country in the multilateral agreement, even if a country in the multilateral agreement has the same number of direct trade partners as a spoke. Additionally, for the aggregate levels of R&D activities I find that the aggregate R&D effort within the multilateral trade agreement exceeds that in the star – the simplest representative of the hub-and-spoke trade system.

Some other findings of the paper concern the change in R&D investments by firms as the network of trade agreements expands. First, consistently with the empirical evidence discussed at the outset, I find that both, in the multilateral and in the regional trade systems, an increase in the number of direct trade partners enhances innovation of a firm, at least as soon as the number of other firms present in the new trade partner countries is not "too large". Secondly, in the multilateral trade agreement, the rate of an increase in firm's R&D effort is declining in the size of the agreement. I show that this result is implied by the decreasing market-enhancement effect of the new trade partners. In addition, for any equal number of direct trade partners of a country in the multilateral trade agreement and of a hub (spoke) in the regional trade system, an increase in the R&D effort caused by a new direct trade partner proves to be smaller for a firm in the multilateral trade system than for a firm in a hub (spoke).

The paper is organized as follows. Section 2 presents the model and describes the two-stage game between firms. Sections 3 and 4 describe the solution of the second and of the first stage of the game, respectively. Section 5 discusses the scale and the competition effects of trade liberalization on firms’ innovation decisions. The joint action of these two effects within the multilateral and the hub-and-spoke trade systems is studied in Scenario 1 and in Scenario 2 of trade liberalization. The scenarios are compared in Section 6 and their policy implications are discussed in Section 7. Finally, Section 8 concludes.

2 The model

Network of regional trade agreements

Consider a setting with N countries where some countries are participants of one or more trade
agreements (TAs) within a certain industry. I model trade agreements between countries with a network: countries are the nodes of the network and each link indicates the existence of a trade agreement between the two linked countries. If two countries have negotiated a TA, then each offers the other a privileged access to its domestic market: the tariffs and restrictions on trade are reduced. Otherwise, for simplicity I assume that tariffs and restrictions on trade between countries which did not sign a TA are trade-prohibitive. So in fact, trade may only exist between countries which have negotiated a TA, that is, only between countries which are directly linked in the network. For any \( i \in 1 : N \), I will denote by \( N_i \) the set of countries with which country \( i \) has a trade link in the network of TAs. These are direct trade partners of \( i \). Let \( |N_i| \) be the cardinality of set \( N_i \). Also, let \( N_i^2 \) be the set of direct trade partners of direct trade partners of \( i \), different from \( i \). In other words, \( N_i^2 \) is the set of two-links-away trade partners of \( i \) in the network of TAs. Notice that some countries may simultaneously be direct and two-links-away trade partners of \( i \). Let \( |N_i^2| \) be the cardinality of set \( N_i^2 \).

This model takes the network of trade agreements as exogenously given. Besides, since the trade agreement between any two countries are reciprocal, all links in the network are undirected and no multiple links exist.

**Demand and cost structure**

In every country, there is a single firm producing one good. The firm in country \( i \) can sell its good in the domestic market and in the markets of those countries with which \( i \) has a trade agreement. Let the output of firm \( i \) (from country \( i \)) produced for consumption in country \( j \) be denoted by \( y_{ij} \). The total output of firm \( i \) is given by \( y_i = \sum_{j\in N_i \cup \{i\}} y_{ij} \). Each firm exporting its good to country \( j \in N_i \cup \{i\} \) faces an inverse linear demand in country \( j \) given by

\[
p_j = a - b \left( y_{ij} + \sum_{k\in N_j \cup \{j\}, k \neq i} y_{kj} \right),
\]

(2.1)

where \( a, b > 0 \) and \( \sum_{k\in N_j \cup \{j\}} y_{kj} \leq a/b \).

Let \( \tau \) denote the trade costs faced by every firm per unit of exports to any of its direct trade partners. These costs include tariffs on unit of export, transportation costs, etc.\(^\text{10}\) The total trade costs faced by firm \( i \) are equal to

\[
t_i(\{y_{ij}\}_{j\in N_i}) = \tau \sum_{j\in N_i} y_{ij}.
\]

(2.2)

\(^9\)This is the degree of \( i \) in the network.
\(^\text{10}\)The analysis carries over in a setting where \( \tau = 0 \). The assumption of zero trade costs is standard in the literature on the formation of the network of TAs. See, for example, Furusawa and Konishi (2007), Goyal and Joshi (2006), and Mauleon et al. (2006).
In addition, each firm can invest in R&D. The R&D effort of the firm helps lower its marginal cost of production. The cost of production of firm $i$ is therefore a function of its production, $y_i$, and the amount of research $x_i$ that it undertakes. I assume that the cost function of each firm is linear and is given by
\[ c_i(y_i, x_i) = (\alpha - x_i)y_i, \]  
where $0 \leq x_i \leq \alpha \ \forall i \in 1 : N$. In the following, I will also assume that $a$ is sufficiently large as compared to $\alpha$ and the costs of trade between countries. Namely, let

**Assumption 1** \[ a > \alpha (1 + \max_{i \in 1:N} |N_i|) + 2\tau. \]

This assumption ensures that the demand for a good is high in all markets, so that in equilibrium, all firms produce strictly positive amounts of both the physical and the technological good. The R&D effort is costly: given the level $x_i \in [0, \alpha]$ of effort, the cost of effort of firm $i$ is
\[ z_i(x_i) = \delta x_i^2, \quad \delta > 0. \]  
Under this specification, the cost of the R&D effort is an increasing function and reflects the existence of diminishing returns to R&D expenditures. The parameter $\delta$ measures the curvature of this function. In the following, it is assumed that $\delta$ is sufficiently large so that the second order conditions hold and equilibria can be characterized in terms of the first-order conditions and are interior.

**Two-stage game**

Firms’ strategies consist of the level of R&D activities and a subsequent production strategy based on their R&D choice. Both strategies are chosen via interaction in a two-stage non-cooperative game. In the first stage, each firm chooses a level of its R&D effort. The R&D effort of a firm determines its marginal cost of production. In the second stage, given these costs of production, firms operate in their domestic market and in the markets of their trade partners by choosing production quantities $\{y_{ij}\}_{i \in 1:N, j \in N_i \cup \{i\}}$ for every market. Each firm chooses the profit-maximizing quantity for each market separately, using the Cournot assumption that the other firms’ outputs are given.

Notice the specific nature of interaction between firms in this game. First, firms compete with each other not in one but in several separate markets. Secondly, since countries trade only with those countries with which they have a trade agreement (a direct link in the network), a firm competes...
of the solution of each firm’s maximization problem. Furthermore, any direct trade partner of firm \( i \) competes with \( i \) in its own market and in the market of firm \( i \), while any two-links-away trade partner of \( i \), who is not simultaneously its direct trade partner, competes with \( i \) only in the market(s) of their common direct trade partner(s). This two-links-away radius of interaction between firms does not mean however that R&D and production choices of firms are not affected by other firms. As soon as the network of TAs is connected, firms that are further than two links away from \( i \) affect R&D and production strategies of \( i \) indirectly, through the impact which they have on R&D and production choices of their own trade partners and trade partners of their partners, etc.

The game is solved using backward induction. Each stage is considered in turn.

## 3 Solving the second stage

In the second stage, each firm \( i \in 1 : N \) chooses a vector of its production plans \( \{ y_{ij} \}_{j \in N_i \cup \{ i \}} \) so as to maximize its profit, conditional on R&D efforts \( \{ x_i \}_{i \in 1:N} \). The profit of firm \( i \) is

\[
\pi_i = \sum_{j \in N_i \cup \{ i \}} \left( a - by_{ij} - b \sum_{k \in N_j \cup \{ j \}, k \neq i} y_{kj} \right) y_{ij} - (\alpha - x_i)y_i - \delta x_i^2 - \tau \sum_{j \in N_i} y_{ij} =
\]

\[
= \sum_{j \in N_i \cup \{ i \}} \left( -by_{ij}^2 - b \sum_{k \in N_j \cup \{ j \}, k \neq i} y_{kj}y_{ij} \right) + (a - \alpha + x_i)y_i - \delta x_i^2 - \tau \sum_{j \in N_i} y_{ij}. \tag{3.1}
\]

Notice that function \( \pi_i \) is additively separable and quadratic in the output levels \( \{ y_{ij} \}_{j \in N_i \cup \{ i \}} \) of firm \( i \). This leads to linear first-order conditions and guarantees the existence and uniqueness of the solution of each firm’s maximization problem.\(^{14}\) Simple algebra results in the Nash-Cournot equilibrium production levels \( \{ y_{ij} \}_{i \in 1:N, j \in N_i \cup \{ i \}} \) of every firm \( i \) for consumption in country \( j \):\(^{15}\)

\[
y_{ii} = \frac{1}{b(|N_i| + 2)} \left( a - \alpha + (|N_i| + 1)x_i - \sum_{j \in N_i} x_j + |N_i|\tau \right), \tag{3.2}
\]

\[
y_{ij} = \frac{1}{b(|N_j| + 2)} \left( a - \alpha + (|N_j| + 1)x_i - \sum_{k \in N_j \cup \{ j \}, k \neq i} x_k - 2\tau \right), \quad i \in 1 : N, j \in N_i. \tag{3.3}
\]

So, the equilibrium output of firm \( i \) in country \( j \in N_i \cup \{ i \} \) is increasing in firm’s own R&D

\[^{13}\text{The network is connected if there exists a path between any pair of nodes.}\]

\[^{14}\text{Since } b > 0, \text{ the second order conditions hold.}\]

\[^{15}\text{Notice that since } x_k \leq \alpha, \]

\[
\sum_{k \in N_j \cup \{ j \}} y_{kj} = \frac{|N_j| + 1}{b(|N_j| + 2)} (a - \alpha) + \frac{1}{b(|N_j| + 2)} \sum_{k \in N_j \cup \{ j \}, k \neq i} x_k - \frac{|N_j|}{b(|N_j| + 2)} \tau \leq \frac{|N_j| + 1}{b(|N_j| + 2)} a - \frac{|N_j|}{b(|N_j| + 2)} \tau < \frac{a}{b}.
\]

In addition, since \( 0 \leq x_k \leq \alpha \) and Assumption 1 holds,

\[
y_{kj} \geq \frac{1}{b(|N_j| + 2)} (a - \alpha - |N_j|\tau) = \frac{1}{b(|N_j| + 2)} (a - (\alpha + |N_j| + 2\tau)) > 0 \quad \forall k \in 1 : N \quad \forall j \in N_k \cup \{ k \}.
\]

\[7\]
To be more precise, for a given network of trade agreements, as soon as $z_i$ is sufficiently steep, the profit function of $i$ is concave in $x_i$. Besides, if $\delta$ is sufficiently high, so that the R&D cost function $z_i$ is sufficiently steep, the profit function of $i$ is concave in $x_i$. To be more precise, for a given network of trade agreements, as soon as

$$\delta > \frac{1}{b} \max_{i \in N} \sum_{j \in N_i \setminus \{i\}} \frac{(|N_j| + 1)^2}{(|N_j| + 2)^2},$$

(4.2)

the second order conditions hold and the profit maximizing R&D efforts of all firms can be found as a solution to the system of linear first-order conditions:

4 Solving the first stage

At the first stage, firms choose R&D efforts. Plugging expressions (3.2)–(3.3) for the output levels \(\{y_{kj}\}_{k \in N_j \cup \{j\}}\) of Cournot competitors in country $j$ into the profit function (3.1) of firm $i$, we obtain the function of the R&D efforts $\{x_k\}_{k \in N_j \cup \{j\}}$. After some calculations, the profit of firm $i$ can be written as

$$\pi_i = \left[ \frac{1}{b} \sum_{j \in N_i \setminus \{i\}} \left( \frac{|N_j| + 1}{(|N_j| + 2)^2} - \delta \right) x_i^2 + \frac{2}{b} (a - \alpha - 2\tau) \sum_{j \in N_i} \frac{|N_j| + 1}{(|N_j| + 2)^2} \right] x_i x_j - \frac{2}{b} \sum_{j \in N_i} \frac{|N_j| + 1}{(|N_j| + 2)^2} x_i x_j \sum_{k \in N_i, k \neq i} \frac{|N_j| + 1}{(|N_j| + 2)^2} x_k + f(\{x_k\}_{k \in N_i \cup N^2_i}),$$ \hspace{1cm} (4.1)

where $f(\{x_k\}_{k \in N_i \cup N^2_i})$ is a function of R&D efforts of $i$’s competitors in different markets which does not distort $i$’s equilibrium effort:

$$f(\{x_k\}_{k \in N_i \cup N^2_i}) = \frac{1}{b} \sum_{j \in N_i} \left( \frac{1}{(|N_j| + 2)^2} a - \alpha - 2\tau - \sum_{k \in N_j \cup \{j\}, k \neq i} x_k \right)^2 + \frac{1}{b} \frac{1}{(|N_i| + 2)^2} \left( a - \alpha + \sum_{j \in N_i} x_j \right)^2.$$
Under Assumption 1, the right-hand side of inequality in Assumption 2 is strictly larger than \( R&D \) expenditures (\( i \) by exerting higher R&D efforts, hold. Moreover, if the inequality in Assumption 2 is strict, solution conditions (4.3)(or (4.4)) is such that 0 is stronger. Together, Assumptions 1 and 2 guarantee that solution investment (\( x \) of \( x \) of \( x \) of \( x \) if \( \delta \) satisfies an additional restriction, stronger than condition (4.2), this solution is ensured to be such that for all \( i \in 1 : N \), 0 < \( x^*_i \leq \alpha \).

\[
\left[-\frac{1}{b} \sum_{j \in N_i \cup \{i\}} \frac{|N_j| + 1}{(|N_j| + 2)^2} + \delta \right] x_i + \frac{1}{b} \sum_{j \in N_i} \left[ \frac{|N_j| + 1}{(|N_j| + 2)^2} + \frac{|N_j| + 1}{(|N_j| + 2)^2} \right] x_j + \frac{1}{b} \sum_{j \in N_i} \sum_{k \in N_j, k \neq i} \frac{|N_j| + 1}{(|N_j| + 2)^2} \sum_{j \in N_i} \frac{|N_j| + 1}{(|N_j| + 2)^2} + \frac{1}{b} (a - \alpha - 2\tau) \sum_{j \in N_i} \frac{|N_j| + 1}{(|N_j| + 2)^2} + \frac{1}{b} (a - \alpha + |N_i|) \frac{|N_i| + 1}{(|N_i| + 2)^2}
\]  
for all \( i \in 1 : N \). In the matrix form, this system can be written as

\[
\Sigma \cdot x = u,
\]  
where \( x \in \mathbb{R}^N \) is a vector of unknowns, \( u \in \mathbb{R}^N \), and \( \Sigma \) is \((N \times N)\) square matrix. As soon as the network of trade agreements is connected, the matrix \( \Sigma \) is generically nonsingular and the right-hand side vector \( u \) is non-zero. Then (4.4) has a unique \textit{generic} solution in \( \mathbb{R}^N \), denoted by \( x^* \). Below it is shown that if \( \delta \) satisfies an additional restriction, stronger than condition (4.2), this solution is ensured to be such that for all \( i \in 1 : N \), 0 < \( x^*_i \leq \alpha \).

First, note that in the \( i \)th first-order condition in (4.3), the value of the expression on the right-hand side and the coefficients multiplying all \( x_k, k \in N_i \cup N_i^2 \cup \{i\} \), are positive. Therefore, the value of \( x_i \) is larger the smaller the values of \( x_j \) and \( x_k \) for all \( j \in N_i \) and \( k \in N_i^2 \). Hence, the condition sufficient for \( x^*_i > 0 \) is that (4.3) evaluated at \( x_j = x_k = \alpha \) \( \forall j \in N_i \), \( k \in N_i^2 \) defines the value of \( x_i \), which is greater than zero. This condition is provided by Assumption 1. Similarly, the sufficient condition for \( x^*_i \leq \alpha \) is that (4.3) evaluated at \( x_j = x_k = 0 \) \( \forall j \in N_i \), \( k \in N_i^2 \) defines \( x_i \), which is smaller than or equal to \( \alpha \). This condition is equivalent to

\textbf{Assumption 2} \quad \delta \geq \frac{1}{ab} \max_{i \in N} \left[ \sum_{j \in N_i} \frac{|N_j| + 1}{(|N_j| + 2)^2} \left( \alpha |N_j| + a - 2\tau \right) + \frac{|N_i| + 1}{(|N_i| + 2)^2} \left( \alpha |N_i| + a + |N_i| \right) \right].

Under Assumption 1, the right-hand side of inequality in Assumption 2 is strictly larger than \( \frac{1}{b} \max_{i \in N} \sum_{j \in N_i \cup \{i\}} \frac{(|N_j| + 1)^2}{(|N_j| + 2)^2} \) from the earlier restriction on \( \delta \) in (4.2). Therefore, Assumption 2 is stronger. Together, Assumptions 1 and 2 guarantee that solution \( x^* \) of a system of the first-order conditions (4.3)(or (4.4)) is such that 0 < \( x^*_i \leq \alpha \) for all \( i \in 1 : N \), and the second order conditions hold. Moreover, if the inequality in Assumption 2 is strict, solution \( x^* \) is interior.\(^\text{16}\)

The specification of the first-order conditions (4.3) suggests that an increase in R&D efforts of \( i \)'s direct and/or two-links-away trade partners trigger a downward shift in \( i \)'s response. Intuitively, by exerting higher R&D efforts, \( i \)'s rivals capture larger shares of the markets and dampen the incentive of \( i \) to invest in R&D. We say that the efforts of firm \( i \) and its direct and two-links-away trade partners are \textit{strategic substitutes} from \( i \)'s perspective.

\(^{16}\)Intuitively, when Assumption 1 holds, the demand for a good in each market is large, which stimulates R&D investment (\( x^*_i > 0 \)). On the other hand, by Assumption 2, the cost of R&D is high, which confines the amount of R&D expenditures (\( x^*_i \leq \alpha \)).
The first-order conditions (4.3) imply that in equilibrium the profit function of firm $i$ is given by

$$
\pi_i = \left[ -\frac{1}{b} \sum_{j \in N_i \cup \{i\}} \frac{(|N_j| + 1)^2}{(|N_j| + 2)^2} + \delta \right] x_i^2 + \frac{1}{b} \sum_{j \in N_i} \frac{1}{(|N_j| + 2)^2} \left( a - \alpha - 2 \tau - \sum_{k \in N_j \cup \{j\}, k \neq i} x_k \right)^2 + \frac{1}{b} \sum_{j \in N_i} \frac{1}{(|N_i| + 2)^2} \left( a - \alpha + |N_i| \tau - \sum_{j \in N_i} x_j \right)^2.
$$

The short proof of this statement is provided in Appendix B.

5 **The impact of trade liberalization on equilibrium R&D efforts**

In the framework of the present model, trade liberalization can be defined as an expansion of the network of trade agreements through an increase in the number of concluded trade agreements (links or links and nodes).

First, consider an impact of trade liberalization on equilibrium R&D efforts of firms in two countries which negotiate a trade agreement with each other. There are two major mechanisms at work. On the one hand, a new trade agreement creates an additional market for each firm (scale effect). This amplifies the return to productivity-enhancing investment, increasing the equilibrium R&D effort of each firm. On the other hand, the new agreement opens the markets of both countries to a new competitor (competition effect). This has two opposite effects on R&D. The enhanced competition dampens the return to R&D through a reduction in the domestic market share of each firm (market share effect of competition); yet, it also increases the return to R&D through a depreciation of markups, which expands the domestic market (markups effect of competition). Thus, overall trade opening between two countries has an ambiguous effect on their equilibrium R&D efforts.

In addition, trade liberalization between any two countries affects R&D decisions of firms in other countries, too. For example, when $i$ and $j$ negotiate a trade agreement, R&D efforts of other direct trade partners of $i$ and $j$ are affected because firms in these countries face higher competition in $i$ and $j$. Then the impact on R&D efforts of the direct trade partners of $i$ and $j$ "spreads" to a larger network: direct trade partners of the direct trade partners of $i$ and $j$ face different R&D efforts and hence, competitive power of their trade partners, which has an impact on their own optimal R&D, etc.

Thus, scale and competition effects of trade liberalization (in any part of the network) can reinforce or dampen the incentive for firms to invest in R&D, the sign and strength of the impact being determined by the precise network structure. In the following section, this issue is addressed in case of multilateral and regional trade liberalization, where each type of liberalization is represented
by the specific type of network.

6 Two scenarios of trade liberalization: Multilateralism versus regionalism

Scenario 1: Symmetric network of trade agreements. Multilateral trade system
Consider a class of symmetric networks of degree \( n \geq 1 \). A symmetric, or regular, network of degree \( n \) is a network where every node has the same number \( n \) of direct contacts. Given the aim of the paper, we are mainly interested in one representative of this class – a complete network. A complete network of degree greater than one represents a multilateral trade system, where all participant countries have a trade agreement with each other and neither country has a trade agreement with a third party. In this framework, an expansion of the multilateral trade system represents a scenario of multilateral trade liberalization. When multilateral trade liberalization involves all world economies, the resulting trade system is a "global free trade".

In addition, the class of symmetric networks comprises a case of one or several simple bilaterals, where every country signs a trade agreement with one and only one other country.\(^{17}\) This case corresponds to the symmetric network of degree one.

\[\begin{align*}
&\frac{-(n+1)^3}{(n+2)^2} + \delta b \right] x + 2n \frac{n+1}{(n+2)^2} x + n \frac{(n+1)(n-1)}{(n+2)^2} x = \frac{(n+1)^2}{(n+2)^2} (a-\alpha) + \frac{n+1}{(n+2)^2} (-2\tau n + \tau n). \\
&x^* = \frac{a - \alpha - \frac{n+1}{n+1}}{-1 + \delta b (1 + \frac{1}{n+1})^2}. \\
\end{align*}\]

\(^{17}\)Up to the early 1990s, trade agreements were, with only a few exceptions, a set of non-intersecting bilateral or "small" multilateral trade agreements (the latter are also called plurilateral RTAs). The source of this evidence is for example, Lloyd and Maclaren (2004).
Now, using this expression for \( x^* \) and formula (4.5) for the equilibrium profit function, we derive the profit of any firm in the symmetric network:

\[
\pi = \left( -\frac{1}{b} \frac{(n+1)^3}{(n+2)^2} + \delta \right) \left( \frac{a - \alpha - \frac{n+1}{n+1} \tau}{-1 + \delta b(1 + \frac{1}{n+1})^2} \right)^2 + \frac{n}{b(n+2)^2} \left( a - \alpha - \frac{n+1}{n+1} \tau - 1 + \delta b(1 + \frac{1}{n+1})^2 \right)^2 + \frac{1}{b(n+2)^2} \left( a - \alpha - n \left( \frac{a - \alpha - \frac{n+1}{n+1} \tau}{-1 + \delta b(1 + \frac{1}{n+1})^2} - \tau \right) \right)^2.
\]

(6.2)

The usual comparative statics analysis leads to the following result:

**Proposition 1** Suppose that Assumptions 1 and 2 hold for all \( n < \bar{n} \), where \( \bar{n} \geq 1 \). Then for any \( n < \bar{n} \), firm’s equilibrium R&D effort \( x^* \) is monotonically increasing in \( n \), while firm’s profit \( \pi \) is monotonically decreasing in \( n \).

Proposition 1 is illustrated with Figure 2, where the equilibrium R&D effort and the profit of a firm in the symmetric network are drawn against the network degree \( n \).

![Figure 2: Equilibrium R&D effort and profit of a country in a symmetric network of degree n](image)

Proposition 1 suggests that multilateral trade liberalization depreciates firms’ profits. However, the incentive for firms to invest in R&D increase. The intuition for this result is easy to grasp. As a new country enters the multilateral trade agreement (or any other agreement which can be represented by the symmetric network), the reduction in the domestic and foreign market shares suffered by each firm is exactly compensated by the participation in the entrant’s market. That is,

---

\[^{18}\text{The simulation is done for the specific parameter values: } \alpha = 7, \ b = 1, \ \bar{n} = 10, \ \text{and } \tau = 2; \ \text{a and } \delta \ \text{fulfill Assumptions 1 and 2.}\]
the negative market share effect of the increased competition is exactly compensated by the positive scale effect associated with access to a new market. As a result, trade liberalization affects R&D only through the reduction in markups – the remaining component of the competition effect. Since the reduction in markups increases the aggregate market size of a firm, the optimal R&D of each firm in the multilateral agreement is increasing in the size of the agreement.

On the other hand, Figure 2 shows that the rates of increase in R&D and decrease in profits are both declining as the number of participant countries (size of the agreement) grows. This observation is implied by the fact that the markups-reducing effect of trade liberalization in the multilateral agreement is declining in the size of the agreement. Basically, using the definition of the demand function in (2.1), I find that the price of the good in each market is given by the decreasing and convex function of $n$:

$$p = \frac{\delta b(n + 2)(a + \alpha(n + 1) + \tau n) - a(n + 1)^2}{-(n + 1)^2 + \delta b(n + 2)^2}.$$ (6.3)

Figure 3: Price on the market of a country in a symmetric network of degree $n$

Furthermore, notice that Proposition 1 allows for the comparison of the equilibrium R&D effort and an individual firm’s profit in the multilateral agreement with those in the bilateral agreement and in autarky. Denote by $x^*_a$ and $\pi_a$ the equilibrium R&D effort and the profit of a firm in autarky, by $x^*_b$ and $\pi_b$, the R&D effort and the profit of a firm in the bilateral agreement, and by $x^*(n)$ and $\pi(n)$, the R&D effort and the profit of a firm in the multilateral agreement of degree $n$ (of size $n + 1$). Then we obtain

**Corollary 1** For any $2 \leq n < \bar{n}$, $x^*_a < x^*_b < x^*(n)$ and $\pi_a > \pi_b > \pi(n)$.

The individual R&D investment of a firm is higher in the multilateral agreement than in the bilateral
agreement and in autarky, while the profit of a firm in multilateral agreement is the lowest.

Finally, for aggregate levels of R&D, Proposition 1 and Corollary 1 imply that the aggregate level of R&D activities within the multilateral trade system is increasing in the size of the system and exceeds the aggregate R&D effort of the same number of countries where each country has negotiated one bilateral trade agreement.

**Scenario 2: Asymmetric network of trade agreements. Hub-and-spoke trade system**

I now examine the case of regional trade liberalization. In the process of regional trade liberalization, some countries (or groups of countries) negotiate one or several bilateral and/or plurilateral agreements with each other. Thus, in contrast to the multilateral type of liberalization considered in Scenario 1, each country may actually be a party to several different trade agreements where other countries do not necessarily have an agreement with each other. As a result, a complex trade system emerges where various regional (preferential) agreements overlap. In the literature, this system is often described as a hub-and-spoke trade system, where some countries (hubs) have a relatively large number of direct trade partners as compared to other countries (spokes), which are mainly involved in trade agreements with hubs.

In this model, I approximate the hub-and-spoke structure by the asymmetric network with two types of nodes – nodes of high degree \( n \) (hubs) and of low degree \( m \) (spokes), \( 1 \leq m < n \). I assume that a fixed positive share of direct trade partners of hubs and spokes is represented by countries of the opposite type. For any hub, other hubs form a share \( 0 \leq \psi < 1 \) of its direct trade partners while a share \( 0 < 1 - \psi \leq 1 \) is represented by spokes. Similarly, for any spoke, other spokes form a share \( 0 \leq \varphi < 1 \) of its direct trade partners and the remaining positive share \( 0 < 1 - \varphi \leq 1 \) is represented by hubs.\(^{19}\) The assumption of fixed (and identical across countries of the same type) shares \( \psi \) and \( \varphi \) significantly simplifies calculations and enriches the comparative statics analysis. For example, it facilitates the study of the effects of variation in the proportion of hubs to spokes among countries’ direct trade partners while the number of these direct trade partners remains unchanged.\(^{20}\)

Some examples of the hub-and-spoke trade system are demonstrated in Table 1.

In any given hub-and-spoke trade system, all hubs exert identical R&D effort \( (x_h) \) and likewise, all spokes exert identical R&D effort \( (x_s) \). Then the system \((4.3)\) of the first-order conditions reduces

\(^{19}\)Notice that in case when \( \psi = 1 \) \( (\varphi = 1) \), we obtain the complete network of degree \( n \) \( (m) \).

\(^{20}\)See Proposition 2.
Table 1: Examples of hub-and-spoke trade system

<table>
<thead>
<tr>
<th>Network characteristics</th>
<th>Network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1: Single star ( (n ) bilaterals of a hub with spokes)</td>
<td><img src="image1" alt="Network Type 1" /></td>
</tr>
<tr>
<td>( n &gt; 1, m = 1, \psi = 0, \varphi = 0 )</td>
<td></td>
</tr>
<tr>
<td>Type 2: Stars with linked hubs</td>
<td><img src="image2" alt="Network Type 2" /></td>
</tr>
<tr>
<td>( n &gt; 1, m = 1, \psi &gt; 0, \varphi = 0 )</td>
<td></td>
</tr>
<tr>
<td>Type 3: Stars sharing spokes</td>
<td><img src="image3" alt="Network Type 3" /></td>
</tr>
<tr>
<td>( n &gt; 1, m &gt; 1, \psi = 0, \varphi = 0 )</td>
<td></td>
</tr>
<tr>
<td>Type 4: Stars with linked hubs, sharing spokes</td>
<td><img src="image4" alt="Network Type 4" /></td>
</tr>
<tr>
<td>( n &gt; 1, m &gt; 1, \psi &gt; 0, \varphi = 0 )</td>
<td></td>
</tr>
<tr>
<td>Type 5: Stars where some spokes are linked with each other</td>
<td><img src="image5" alt="Network Type 5" /></td>
</tr>
<tr>
<td>( n &gt; 1, m &gt; 1, \psi = 0, \varphi &gt; 0 )</td>
<td></td>
</tr>
</tbody>
</table>

Remark: Red nodes stand for hubs, green nodes stand for spokes.

to two equations:

\[
\begin{align*}
\left( - (1 - \psi) n \frac{(m + 1)^2}{(n + 2)^2} - (n \psi + 1) \frac{(n + 1)^2}{(n + 2)^2} + \delta b + \psi n \frac{2(n + 1)}{(n + 2)^2} + (1 - \psi) n \frac{m + 1}{(m + 2)^2} (1 - \varphi) m - 1 \right) \\
+ \psi n \frac{n + 1}{(n + 2)^2} (n \psi - 1) \cdot x_n + (1 - \psi) n \left( \frac{n + 1}{(n + 2)^2} + \frac{m + 1}{(m + 2)^2} + \frac{n + 1}{(n + 2)^2} n \psi + \frac{m + 1}{(m + 2)^2} \varphi_m \right) \cdot x_s
\end{align*}
\]

\[
\begin{align*}
(a - \alpha - 2 \tau) \left( (1 - \psi) n \frac{m + 1}{(m + 2)^2} + \psi n \frac{n + 1}{(n + 2)^2} \right) + \frac{n + 1}{(n + 2)^2} (a - \alpha + n \tau) \quad (6.4)
\end{align*}
\]

\[
\begin{align*}
\left( - (1 - \varphi) m \frac{(n + 1)^2}{(n + 2)^2} - (\varphi m + 1) \frac{(m + 1)^2}{(m + 2)^2} + \delta b + \varphi m \frac{2(m + 1)}{(m + 2)^2} + (1 - \varphi) m \frac{n + 1}{(n + 2)^2} (1 - \psi) n - 1 \right) \\
+ \varphi m \frac{m + 1}{(m + 2)^2} (\varphi m - 1) \cdot x_s + (1 - \varphi) m \left( \frac{n + 1}{(n + 2)^2} + \frac{m + 1}{(m + 2)^2} + \frac{n + 1}{(n + 2)^2} n \psi + \frac{m + 1}{(m + 2)^2} \varphi_m \right) \cdot x_h
\end{align*}
\]

\[
\begin{align*}
(a - \alpha - 2 \tau) \left( (1 - \varphi) m \frac{n + 1}{(n + 2)^2} + \varphi m \frac{m + 1}{(m + 2)^2} \right) + \frac{m + 1}{(m + 2)^2} (a - \alpha + m \tau) \quad (6.5)
\end{align*}
\]

These equations uniquely identify equilibrium R&D efforts of a hub and a spoke. Extensive calculations lead to the closed-form solution \((x^*_h, x^*_s)\), which has a cumbersome representation and therefore, is left for the Appendix.\(^{21}\)

\(^{21}\)See the proof of Proposition 2 in Appendix B.
Consider the impact of trade liberalization in the hub-and-spoke trade system on equilibrium R&D efforts $x_h^*, x_s^*$. In contrast to the case with the multilateral trade system, in the asymmetric hub-and-spoke structure, the negative component of the competition effect of trade liberalization (market share effect of competition) is generally not compensated by the positive scale effect. Therefore, a priori the impact of trade liberalization on R&D in the hub-and-spoke system is ambiguous. However, the comparative statics of $x_h^*$ and $x_s^*$ reveal some insights about the effects of an expansion/variation of the hub-and-spoke trade structure on the R&D effort of a firm.

**Proposition 2** Suppose that Assumptions 1 and 2 hold for all $m < n \leq \bar{n}$, where $\bar{n} > 1$. Then there exists $\Delta > 0$ such that for any $\delta \geq \Delta$ and for any $m < n < \bar{n}$, the following statements are fulfilled:

1. the equilibrium R&D effort $x_h^*$ of a hub is monotonically increasing in $n$ and monotonically decreasing in $m$ and in $\psi$;

2. the equilibrium R&D effort $x_s^*$ of a spoke is monotonically decreasing in $n$ and monotonically increasing in $\varphi$. Furthermore, $x_s^*$ is monotonically increasing in $m$ if at least one of the conditions holds:

   (a) the trade costs are sufficiently high: $\tau \geq \frac{1-\varphi}{3-2\varphi}(a-\alpha)$;

   (b) the share of other spokes among direct trade partners is at least $1/3$: $\varphi \geq \frac{1}{3}$;

   (c) the gap between $n$ and $m$ is relatively small: $n \leq m^2$, that is, $1 < \frac{n}{m} \leq m$.

Proposition 2 states that as soon as the specified parameter restrictions hold (including conditions (a) – (c)), the equilibrium R&D effort of a hub (spoke) is increasing in the number of its direct trade partners but is decreasing in the number of direct trade partners of spokes (hubs). In addition, for both a hub and a spoke, the higher the share of hubs among their direct trade partners, the lower the optimal R&D effort. Thus, the larger the number of directly accessible markets and the lower the number of competitors in these markets, the higher the incentive for firms to innovate. This observation suggests that, in the hub-and-spoke trade system, the competition effect of trade liberalization on firm’s R&D is negative but the positive scale effect of any direct trade partner dominates its negative competition effect.

The negative impact of competition on equilibrium R&D decisions of spokes sheds light on conditions (a) – (c), which guarantee that an increase in $m$ enhances spokes’ R&D investments. Recall that the specification of a hub-and-spoke trade system in this model is such that an increase in the number of a spoke’s direct trade partners $m$ is associated with an increase in the number of
both types of partners – hubs and spokes.22 Since the market of a hub is ”small” – smaller than the market of a spoke, an increase in the spoke’s foreign market share may actually be smaller than a decrease in the share of the domestic market. As a result, the positive scale effect of an increase in \( m \) on R&D investment of a spoke may be dominated by the negative competition effect. Conditions (a) – (c) ensure that this would not be the case if: (a) the trade costs of firms are sufficiently high to restrict the amount of exports from new trade partners, (b) hubs represent only a minor share of direct trade partners of a spoke, or alternatively, (c) the number \( m \) of competitors in the spoke’s market is comparable to \( n \), so that the loss in the domestic market share of the spoke is not larger than the gain in the market of a new hub market.

The results of Proposition 2 are illustrated with Figures 5 and 6 in Appendix C.23

**Comparison of multilateral and regional trade systems**

In this section I examine how the impacts of different types of trade liberalization compare in terms of R&D investments of firms. The comparison is made in two steps. First, I investigate the basic relationship between the R&D level of a firm in the multilateral trade agreement and a firm in the regional, hub-and-spoke trade system. After that I distinguish between different types of the hub-and-spoke system and study the ranking of R&D efforts of firms across various types of the hub-and-spoke system and the multilateral system. Consider each step in turn.

**Step 1** To gain some insights about the sources of variation in R&D efforts of a firm across the multilateral and the regional, hub-and-spoke trade systems, let us first assume that the demand/price for the good is the same across markets, and that all firms operating in the market of a country obtain the same share of the market. Then, given the fixed number of direct trade partners, it is purely the number of other firms/competitors present in each trade partner country that determines the aggregate market size of a firm. The fewer competitors, the larger the aggregate market size of a firm and the higher the return to R&D investment.

In this simplified framework, for any number \( n \) of direct trade partners, the aggregate market size of a hub in any hub-and-spoke trade system is larger than that of a firm in the multilateral agreement. The opposite is true for spokes. For any number \( m \) of direct trade partners, the total market size of a spoke is smaller than that of a country in the multilateral agreement. To clarify the first statement, observe that while in the multilateral agreement (of degree \( n \)), the number of competitors of a firm is \( n \) in each of its \( n \) foreign markets, in the hub-and-spoke trade system, the number of competitors of a firm is \( n \) only in \( \psi \cdot n \) of its foreign markets and it is less than \( n \) in the

---

22The proportion of spokes to hubs among the new trade partners is determined by \( \varphi \): the lower \( \varphi \), the higher the relative number of hubs.

23Both figures are produced using the same parameter values as for Figure 2 in Scenario 1. In addition, for Figure 5, I set \( \psi = \varphi = 0 \) and for Figure 6, \( n = 6 \), \( m = 2 \).
remaining markets. A similar argument is applicable for spokes.

Recall that these conclusions are derived under the assumption of equal demand and equal market shares of firms in every market. But in fact, they hold without this assumption. Formally, the result is an immediate implication of Proposition 2 and the short proof is provided in Appendix B:

**Proposition 3** For any $0 \leq \psi, \varphi < 1$ and for any $n, m > 1$ such that $n > m$,

\[
x_h^* > x^*(n) > x^*(m) > x_s^*.
\]

Moreover, the same inequalities hold when a hub and a spoke belong not just to one, but to any different types of the hub-and-spoke structure.\(^{24}\)

**Step 2** Now, I compare equilibrium R&D efforts of firms across various types of the hub-and-spoke trade structure. To that end, I restrict attention to the specific types of the hub-and-spoke structure presented in Table 1. Notice that by Proposition 3, it only remains to compare separately R&D efforts of hubs and R&D efforts of spokes, since R&D of a hub is always higher than R&D of a spoke both in one and in different types of the hub-and-spoke structure.

As before, assume that the demand for the good is the same in all markets and that all firms (hubs and spokes) share each market equally. Consider the differences in market sizes of hubs and spokes across various hub-and-spoke structures. With regard to hubs, observe that for any number, $n$, of a hub’s direct trade partners, a hub in the star (Type 1 system) enjoys the lowest competition in any of its foreign markets as compared to hubs in the other systems. Therefore, a hub in the star has the largest total market size. As a number of firms (competitors) in markets of a hub’s direct trade partners grows, the aggregate market size of the hub decreases. This is the case when either the number of a spoke’s direct trade partners, $m$, grows (Type 3 system), the share of hubs among direct trade partners, $\psi$, increases (Type 2 system) or when both changes in $m$ and $\psi$ happen simultaneously (Type 4 system). Furthermore, the larger the increase in $m$ and/or $\psi$, the smaller the size of a hub’s aggregate market.

For spokes the situation is symmetric. Given any number, $n$, of a hub’s direct trade partners, a spoke in the star (Type 1 system) has access to a single foreign market ($m = 1$). Therefore, a spoke’s market in the star is smaller than a spoke’s market in any other hub-and-spoke trade system.\(^{25}\) As the number of direct trade partners of a spoke, $m$, increases (Type 3 system), the market of a spoke expands. It expands even further if the share of spokes among direct trade partners, $\varphi$, grows (Type

\(^{24}\)Recall from Scenario 1 that $x^*(n)$ denotes the equilibrium R&D effort of a firm in the multilateral agreement of degree $n$.

\(^{25}\)In fact, on the assumption of equal demand and equal market shares of firms in every market, the market size of a spoke in Type 2 system is the same as in the star.
5 system). Moreover, the larger the increase in \(m\) and/or \(\varphi\), the larger the aggregate market size of a spoke.

As in Step 1, the insights gained on the assumption of equal demand and equal market shares of firms in every market prove to be valid when the assumption is relaxed. This leads to Proposition 4, which is formally derived in Appendix B. To state the proposition, I denote by \(x^*_h\) the equilibrium R&D effort of a hub and by \(x^*_s\), the equilibrium R&D effort of a spoke in the hub-and-spoke trade system of Type \(i\), \(i \in 1:5\).

**Proposition 4** Consider Types 1–5 of the hub-and-spoke trade structure. Suppose that (i) \(n\) is the same across all types, (ii) \(m\) is the same across all types where \(m > 1\) (Types 3, 4 and 5), and (iii) \(\psi\) is the same across all types where \(\psi > 0\) (Types 2 and 4). Let \(x^*(n)\) and \(x^*(m)\) be defined for \(n\) and \(m > 1\), identical to those in Types 1 – 5 of the hub-and-spoke structure. Then firms’ equilibrium R&D efforts in Types 1–5 of the hub-and-spoke structure and in the multilateral agreement rank as follows:

\[
x^*_h > x^*_s > x^*_h > x^*(n) > x^*(m) > x^*_s > x^*_s > x^*_s.
\]

With respect to the equilibrium R&D effort \(x^*_h\) of a hub and \(x^*_s\) of a spoke in Type 2 system, the following inequalities hold:

\[
x^*_h > x^*_h > x^*_h \quad \text{and} \quad x^*_s > x^*_s.
\]

Proposition 4 is illustrated with Figure 4 and Figure 7.

Thus, the R&D efforts of firms in the multilateral system and in various types of the hub-and-spoke trade systems vary substantially. The highest R&D incentives exist for a hub, especially for a hub in the star (Type 1 system), whereas for a spoke in the star the incentives are the lowest. As the number of direct trade partners of a spoke and/or the share of spokes (hubs) among direct trade partners of each spoke (hub) increase, the levels of R&D investment of hubs and spokes converge. They coincide at the level of R&D investment of a firm in the multilateral agreement, which therefore, takes an average position: it is lower than R&D of a hub but higher than R&D of a spoke.

Further to comparing individual R&D investments by firms, I compare the aggregate levels of R&D activities of the same total number of countries in the multilateral trade agreement and in the star – bilateral agreements of one country (a hub) with the others (spokes). I find that although the individual R&D effort of a hub in the star is much higher than the R&D effort of a single country in the multilateral agreement, the aggregate R&D in the star is lower than in the multilateral agreement. This observation is demonstrated by Figure 8 in Appendix C.

Finally, Figures 4 and 7 provide some insights concerning the rates of an increase in firms’
Figure 4: Equilibrium R&D efforts in the multilateral and in the hub-and-spoke trade system as a function of $n$ (the upper sub-figure) and as a function of $m$ (the lower sub-figure).
individual R&D efforts as the network of trade agreements expands. They show that under the conditions of Proposition 2 (parameter restrictions (9.6)–(6.8) and conditions (a)–(c)), for any equal number of direct trade partners of a country in the multilateral trade agreement and of a hub (spoke) in the hub-and-spoke trade system, an increase in the R&D effort caused by a new direct trade partner is smaller for a firm in the multilateral trade system than for a hub (spoke).

7 Policy implications

The previous analysis suggests that the structure of the network of trade agreements and the position of a country in this network are key for understanding the differences in R&D and productivity levels of firms across the multilateral and the regional types of trade systems. This feeds into the ongoing debate on gains and losses of multilateralism versus regionalism, especially with respect to the intensive proliferation of regional trade agreements among the WTO member countries. The paper suggests that even though both, the multilateral and the bilateral types of trade liberalization improve R&D and productivity of a country in autarky, the comparative productivity gains of regionalism depend heavily on the number of trade agreements signed by the country. Countries which negotiate sufficiently many regional trade agreements – ”strong negotiators” – improve their R&D and productivity more in the regional trade system than in the multilateral system. At the same time, the other countries – ”weak negotiators” – obtain unambiguously higher productivity gains in the multilateral trade system. The latter observation suggests that from a perspective of a small economy, which normally becomes a spoke in the regional trade system, the prospects for productivity improvements within the multilateral trade system are generally better than within the regional system.26

In addition, the paper indicates the positive impact of the expansion of the WTO on R&D and productivity of every member country. In the same manner, the consolidation of several smaller plurilateral blocks or their accession to the WTO enhances R&D in every country.27

Recall that for the specific classes of networks considered in Scenario 1 and in Scenario 2, the equilibrium R&D effort of a firm is increasing in the number of its direct trade partners (scale and competition effects of trade liberalization) but decreasing in the number of its two-links-away trade

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26The finding of a substantially lower R&D and productivity levels in a spoke economy as compared to those in a hub and in a country within the multilateral system, supports the argument of earlier studies about the disadvantageous position of spokes. For instance, Baldwin (2003), Kowalczyk and Wonnacott (1992), Deltas et al. (2005), Lloyd and Maclaren (2004), and De Benedictis et al. (2005) find that welfare and income levels are lower for spokes than for hubs and than for countries in the complete network.

27According to Fiorentino et al. (2007), the number of merging regional trade agreements is currently increasing. Examples include EC-GCC, SACU-MERCOSUR, among others.
partners (competition effect only). In Appendix A, I further investigate the issue of the impact which direct and two-links-away trade partners have on R&D of a firm in a generic network under the assumption of small external effects. Consistently with the results of the previous analysis, I find that new direct trade partners of a firm (mostly) increase its R&D investment and the smaller the number of competitors in the new markets, the larger the increase in R&D.

8 Conclusion

This paper develops a model of international trade with firm-level productivity improvements via R&D. Firms in different countries sell their product and compete in a Cournot fashion with other firms in the domestic market and in the markets of their trade partners. The trade partners of any country/firm are defined by the network of trade agreements: countries which are linked in the network are (direct) trade partners of each other.

I focus on two specific types of networks: the complete and the hub-and-spoke network. In the model, these networks represent trade arrangements which arise as a result of multilateral or regional trade liberalization, respectively. I study how the structure of the trade arrangement and the position of a country in this structure affect R&D investments by firms. In this manner I address the issue of the difference in the impact of multilateral and regional types of trade liberalization on firms’ R&D and productivity.

I show that the R&D response of firms to trade liberalization is the net outcome of two different effects: one, stimulating R&D through the creation of new markets (scale effect), and the other, deterring or improving R&D through the emergence of new competitors (competition effect). I find that the sizes of both effects vary across the multilateral and the regional/hub-and-spoke trade systems. Basically, a new country entering the multilateral or the hub-and-spoke trade system represents different ”value added” for firms in every system since the effective size of the new market and the competitive impact of the new firm depend on the structure of the trade system.

The variations in the scale effect and the competition effect of trade across structures leads to variations in firms’ aggregate market sizes. In turn, the difference in firms’ market sizes explains the difference in levels of firms’ R&D investments. For the same number of direct trade partners, the R&D effort of a hub in the hub-and-spoke trade system is higher than the R&D effort of a country in the multilateral agreement. On the other hand, R&D of a spoke is lower than R&D of a hub and lower than R&D of a country in the multilateral agreement, even if a country in the multilateral agreement has the same number of direct trade partners as a spoke.

In addition, consistently with the empirical evidence, I find that a new market opening increases
R&D of a firm in the multilateral system and for the most part, it increases R&D of a firm in the regional system, too. However, the size of an increase in R&D varies across different types of trade systems. I show that for any equal number of direct trade partners of a country in the multilateral trade agreement and of a hub (spoke) in the hub-and-spoke trade system, the growth of the R&D effort caused by a new direct trade partner is lower for a firm in the multilateral trade system than for a hub (spoke) in the regional system.

The paper suggests some policy implications. For example, with regard to benefits and losses of regionalism versus multilateralism, the paper indicates that the regional trade liberalization is likely to be more beneficial than the multilateral trade liberalization for R&D and productivity level of ”strong negotiators” (hubs) – countries which sign sufficiently many regional trade agreements. At the same time, R&D and productivity level of weaker negotiators are higher if the countries choose the multilateralist alternative.

Lastly, it is important to emphasize that the results of this paper are driven purely by the scale effect and the competition effect of trade liberalization, which are in turn determined by the structural characteristics of trade agreements. In order to precisely isolate the impact of the structure of trade agreements, the model abstracts from other channels through which trade liberalization may affect R&D. For example, the excluded channels are the R&D spillover effect and the firms selection effect of trade. Additionally, for the sake of simplicity, the model disregards important inequalities in terms of geographical size and initial income/resources across countries. These limitations of the paper suggest directions for further research. In particular, the empirical test of the model could provide more insights.

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28 Recall that in the regional hub-and-spoke trade system, R&D of a spoke declines in response to a new market opening if the negative competition effect on R&D of the spoke’s new trade partners outweighs the positive scale effect. However, by simulating the model for the star network under various parameter assumptions, I find that opening trade with the hub decreases R&D of a spoke when the number of competitors in the hub’s market is ”unrealistically high” (more than 100).

29 As suggested by Melitz (2003), the firms selection effect results from the initial difference in productivity of multiple domestic firms in each country and from the existence of the fixed costs of production and exporting.
9 Appendix

Appendix A: Equilibrium R&D efforts in arbitrary network. The case of small external effects

Consider the system of first-order optimality conditions (4.3). Below I study properties of the solution to this system when the magnitude of local effects – effects of interaction between firms in the network – is arbitrarily small. I seek the ranking of optimal R&D decisions of firms in accordance with simple characteristics of firms’ positions in the network, such as the nodal degrees and the sum of neighbors’ degrees. To derive this ranking, I employ the asymptotic approach suggested by Bloch and Quérou (2008).30

Notice that the system of linear first-order conditions (4.3) can be written as

\[
\delta x_i - \frac{1}{b} \left[ \sum_{j \in N_i \cup \{i\}} \frac{(|N_j| + 1)^2}{(|N_j| + 2)^2} x_j - \sum_{j \in N_i} \left( \frac{|N_i| + 1}{(|N_i| + 2)^2} + \frac{|N_j| + 1}{(|N_j| + 2)^2} \right) x_j - \sum_{j \in N_i} \sum_{k \in N_j \setminus k} \frac{|N_j| + 1}{(|N_j| + 2)^2} x_k \right] = \frac{1}{b} \left( a - \alpha - 2\tau \right) \sum_{j \in N_i} \frac{|N_j| + 1}{(|N_j| + 2)^2} x_j + \frac{1}{b} \left( a - \alpha + |N_i|\tau \right) \frac{|N_i| + 1}{(|N_i| + 2)^2}, \quad i \in 1 : N.
\]

In the matrix form this has a simple representation

\[
\left( \delta I - \frac{1}{b} B \right) \cdot x = \frac{1}{b} \tilde{u}.
\]

Alternatively,

\[
(I - \lambda B) \cdot x = \lambda \tilde{u}, \quad (9.1)
\]

where \( \lambda = \frac{1}{55} \). In this system, matrix \( \lambda B \) is the matrix of local effects. Below, I investigate the solution to (9.1) when the norm of matrix \( \lambda B \) capturing the magnitude of local effects is small.

First, following Bloch and Quérou (2008), I define a vector sequence \( f = (c^1, c^2, \ldots, c^m, \ldots) \), where each vector \( c^m \) is given by

\[
c^m = \lambda^m \tilde{u} B^{m-1}.
\]

The first terms of this sequence are

\[
c^1 = \lambda \tilde{u},
\]

\[
c^2 = \lambda^2 \tilde{u} B,
\]

\[
c^3 = \lambda^3 \tilde{u} B^2.
\]

Using the sequence \( f \), I can now state the approximation result, which provides an equivalence

---

30 As emphasized in Bloch and Quérou (2008), at least two arguments can defend the usefulness of studying network effects whose magnitude is small. First, when the matrix of interactions is complex, this may be the only way to evaluate the equilibrium R&D decisions for an arbitrary network structure. Secondly, by continuity, the insights obtained for small external effects continue to hold as the magnitude of externalities increases.
between the ranking of the components of the solution $\mathbf{x}^*$ to (9.1) and the lexicographic ordering of the components of $\mathbf{f}$ when the magnitude of $\|\lambda\mathbf{B}\|$ is close to zero.

**Proposition 5** Consider a system of linear equations (9.1). Suppose that $\|\lambda\mathbf{B}\|$ is sufficiently small\(^{31}\) for a given $0 < \varepsilon < 1$, $\|\lambda\mathbf{B}\| \leq \frac{\varepsilon}{N}$. Then there exists a unique solution $\mathbf{x}^*$ and $K > 1$ such that for any $i, j \in 1 : N, i \neq j$,

$$|x_i^* - x_j^* - (c_i^M - c_j^M)| \leq \lambda \cdot \frac{\varepsilon^{K+1}}{1 - \varepsilon} \cdot 2\|\tilde{\mathbf{u}}\|,$$

where $(c_M)_i$ and $(c_M)_j$ are the first unequal elements of the sequences $\mathbf{f}_i = (c_{i,1}, c_{i,2}, \ldots, c_{i,m}, \ldots)$ and $\mathbf{f}_j = (c_{j,1}, c_{j,2}, \ldots, c_{j,m}, \ldots)$: $c_i^M \neq c_j^M$ and $c_i^m = c_j^m$ for all $m < M$.

Thus, if the upper bound for the magnitude of local effects is close to zero,

$$x_i^* > x_j^* \Leftrightarrow \mathbf{f}_i \succ \mathbf{f}_j,$$

where $\mathbf{f}_i \succ \mathbf{f}_j$ stands for lexicographic dominance of $\mathbf{f}_i$ over $\mathbf{f}_j$. This means that in order to compare equilibrium R&D efforts of different firms, one can restrict attention to the first order term $\mathbf{c}^1$, or if the first order terms are equal, to the second order term $\mathbf{c}^2$, etc. As a result, the ranking of optimal R&D choices of firms reduces to the ranking of characteristics of firms’ positions in the network.

Consider a pair of firms $(i, i')$, $i, i' \in 1 : N$, such that $\tilde{\mathbf{u}}_i \neq \tilde{\mathbf{u}}_{i'}$. Then by Proposition 5, if

$$\|\lambda\mathbf{B}\| \leq \frac{\varepsilon}{N}$$

for some $0 < \varepsilon < 1$, then the difference between $x_i^*$ and $x_{i'}^*$ can be approximated by the difference between $\tilde{\mathbf{u}}_i$ and $\tilde{\mathbf{u}}_{i'}$ such that the measurement error does not exceed $\lambda \cdot \frac{\varepsilon^{K+1}}{1 - \varepsilon} \cdot 2\|\tilde{\mathbf{u}}\|$, where

$$\|\tilde{\mathbf{u}}\| = \max_i (a - \alpha - 2\tau) \sum_{j \in N_i} \frac{|N_j| + 1}{(|N_j| + 2)^2} + (a - \alpha + |N_i|\tau) \frac{|N_i| + 1}{(|N_i| + 2)^2}.$$  

So, when the local effects are small, the R&D effort chosen by firm $i$ is at least as high as the R&D effort of firm $i'$ if and only if

$$(a - \alpha - 2\tau) \sum_{j \in N_i} \frac{|N_j| + 1}{(|N_j| + 2)^2} + (a - \alpha + |N_i|\tau) \frac{|N_i| + 1}{(|N_i| + 2)^2} \geq (9.2)$$

$$\geq (a - \alpha - 2\tau) \sum_{j \in N_{i'}} \frac{|N_j| + 1}{(|N_j| + 2)^2} + (a - \alpha + |N_{i'}|\tau) \frac{|N_{i'}| + 1}{(|N_{i'}| + 2)^2}.$$  

The inequality (9.2) suggests that at the first order the R&D effort of firm $i$ is decreasing in the number $|N_j|$, $j \in N_i$, of $i$’s two-links-away trade partners. In addition, the R&D effort of firm $i$ is increasing in the number $|N_i|$ of $i$’s direct trade partners as soon as the new trade partner $j'$ is such

\(^{31}\)As in Bloch and Quérou (2008), I use the $l_\infty$ vector norm defined by $\|\mathbf{A}\| = \max_{i,j} |a_{ij}|$.
that
\[
\frac{|N_i| + 2}{(|N_i| + 3)^2} (a - \alpha + (|N_i| + 1)\tau) - \frac{|N_i| + 1}{(|N_i| + 2)^2} (a - \alpha + |N_i|\tau) + (a - \alpha - 2\tau) \frac{|N_j| + 1}{(|N_j| + 2)^2} > 0.
\]

Alternatively, this can be written as
\[
(a - \alpha - 2\tau) \frac{|N_j| + 1}{(|N_j| + 2)^2} > \frac{|N_i|^2 + 3|N_i| + 1}{(|N_i| + 2)^2} (a - \alpha + |N_i|\tau) - \frac{|N_i| + 2}{(|N_i| + 3)^2} \tau.
\] (9.3)

It is easy to see that under Assumption 1 the left-hand side of inequality (9.3) is decreasing in |N_j|. This means that the additional direct trade partner j of i increases i’s incentives to innovate as soon as the number of competitors |N_j| of i in market j is sufficiently low. Thus, in accordance with the earlier discussion in the paper, opening trade with an additional trade partner increases the R&D investment of firm i if the actual market size of the new trade partner is large enough.

The finding of a positive effect of direct trade partners and a negative effect of two-links-away trade partners of i on i’s equilibrium R&D effort, together with the conditions which guarantee these effects, are consistent with the findings of Scenarios 1 and 2 discussed in Section 6.

Appendix B: Proofs

Derivation of the profit function in (4.5)
The profit function in (4.1) can be written as
\[
\pi_i = 2x_i^* \left[ \frac{1}{b} (a - \alpha - 2\tau) \sum_{j \in N_i} \frac{|N_j| + 1}{(|N_j| + 2)^2} + \frac{1}{b} (a - \alpha + |N_i|\tau) \frac{|N_i| + 1}{(|N_i| + 2)^2} - \frac{1}{b} \sum_{j \in N_i} \left( \frac{|N_j| + 1}{(|N_j| + 2)^2} \right) x_j^* - \frac{1}{b} \sum_{j \in N_i} \sum_{k \in N_j \& k \neq i} \frac{|N_j| + 1}{(|N_j| + 2)^2} x_k^* \right] - \left[ - \frac{1}{b} \sum_{j \in N_i \cup \{i\}} \frac{(|N_j| + 1)^2}{(|N_j| + 2)^2} + \delta \right] x_i^* + f(\{x_k\}_{k \in N_i \cup N_i^2}).
\]

By the first-order conditions (4.3), this reduces to
\[
\pi_i = 2 \left[ - \frac{1}{b} \sum_{j \in N_i \cup \{i\}} \frac{(|N_j| + 1)^2}{(|N_j| + 2)^2} + \delta \right] x_i^* \quad + \quad f(\{x_k\}_{k \in N_i \cup N_i^2}) = \left[ - \frac{1}{b} \sum_{j \in N_i \cup \{i\}} \frac{(|N_j| + 1)^2}{(|N_j| + 2)^2} + \delta \right] x_i^* \quad + \quad \frac{1}{b} \sum_{j \in N_i} \left( \frac{1}{(|N_j| + 2)^2} \right) \left( a - \alpha - 2\tau - \sum_{k \in N_i \cup \{j\}, k \neq i} x_k^* \right)^2 \quad + \quad \frac{1}{b} \left( \frac{1}{(|N_i| + 2)^2} \right) \left( a - \alpha + |N_i|\tau - \sum_{j \in N_i} x_j^* \right)^2.
\]
Proof of Proposition 1

First, notice that in case of a symmetric network of degree \( n \), the right-hand side of inequality in Assumption 1 is an increasing function of \( n \) and also the right-hand side of inequality in Assumption 2 is an increasing function of \( n \), provided that Assumption 1 holds. Therefore, for Assumptions 1 and 2 to be fulfilled for all \( n < \bar{n} \), it is enough to ensure that these assumptions hold for \( n = \bar{n} \). The resulting restrictions are

\[
a > \alpha(1 + \bar{n}) + 2\tau, \quad \text{and} \quad \delta \geq \frac{1}{\alpha b (\bar{n} + 2)^2} \left( (\alpha \bar{n} + a)(\bar{n} + 1) - \tau \bar{n} \right). \tag{9.4}
\]

\[
\delta \geq \frac{1}{\alpha b (\bar{n} + 2)^2} \left( (\alpha \bar{n} + a)(\bar{n} + 1) - \tau \bar{n} \right). \tag{9.5}
\]

The proof of Proposition 1 is established in two steps.

1. R&D effort \( x^* \) is monotonically increasing in \( n \)

Taking a derivative of \( x^* \) in (6.1) with respect to \( n \), we obtain:

\[
\frac{\partial x^*}{\partial n} = \frac{\tau}{(n+1)²} \left( \frac{1}{(n+1)^2} \right) \left( -1 + \delta b \left( 1 + \frac{1}{n+1} \right)^2 \right) + 2\delta b \left( 1 + \frac{1}{n+1} \right) \left( a - \alpha - \frac{n}{n+1} \tau \right) = \frac{1}{(n+1)^2} \left( \tau + \delta b \left( 1 + \frac{1}{n+1} \right) \left( -\tau (1 + \frac{1}{n+1}) + 2 (a - \alpha - \frac{n}{n+1} \tau) \right) \right). \]

The sign of this derivative is positive as soon as

\[
2(a - \alpha - \frac{n}{n+1} \tau) > \tau (1 + \frac{1}{n+1}).
\]

One can readily see that this inequality holds due to the restriction on \( a \) in (9.4).

2. profit \( \pi \) is monotonically decreasing in \( n \)

Due to the computational complexity, I present only a schematic proof of this statement.

Taking the derivative of \( \pi \) in (6.2) with respect to \( n \), we obtain the expression represented by the product of the ratio \( \frac{1}{(2n^4 - 4bn^3 + 4b^2\delta n^2 - b^3\delta^2 + b^2\delta + 1)^3} \) and the quadratic polynomial of \( \tau \). The ratio is negative for any \( n \geq 1 \) due to the restriction on \( \delta \) in (9.5). On the other hand, the value of the polynomial is positive for any \( n \geq 1 \) as soon as parameters satisfy the restrictions (9.4) and (9.5). The latter is established via two steps.

- First, I find that due to the restriction (9.5) the coefficient of the polynomial at the quadratic term \( \tau^2 \) is negative for any given \( n \geq 1 \). Besides, the constant term is positive.
  Hence, the graph of the quadratic function is a parabola with downward-directed branches and two real roots – one positive and one negative.
- Since the unit trade cost \( \tau \) is positive and by the restriction (9.4), it does not exceed
Taking a derivative of \( h \), to establish that the value of the polynomial is positive for all \( \tau \in (0, \frac{1}{2}(a - \alpha)) \), it suffices to show that the value of the polynomial is positive at \( \tau := \frac{1}{2}(a - \alpha) \). One can find that this is indeed the case, provided that (9.4) and (9.5) hold.

Thus, for all \( n \geq 1 \) and any parameter values satisfying the conditions (9.4) and (9.5), the derivative of \( \pi \) with respect to \( n \) is negative, so that the profit function is decreasing in \( n \).

\[ \]

**Sketch of the proof of Proposition 2**

Notice that to ensure that Assumptions 1 and 2 hold for all \( m < n \), it is enough to impose the restrictions

\[ a > \alpha (1 + \bar{n}) + 2\tau, \quad \text{and} \quad \delta \geq \frac{1}{\alpha b} \left[ \frac{\bar{n} + 1}{(\bar{n} + 2)^2} \bar{n} (\alpha \bar{n} + a - 2\tau) + \frac{2}{9} ((\alpha \bar{n} + a)(\bar{n} + 1) - \tau \bar{n}) \right]. \]

(9.6)

(9.7)

The equilibrium R&D effort of a hub and a spoke are given by the solution to the system of equations (6.4) – (6.5):

\[
x^*_h = \frac{(a - \alpha - 2\tau)[(n\psi(n + 1)(m + 2) + (m + 1)(n + 2)^2]((a - \alpha - 2\tau)\varphi m + a - \alpha + m\tau) - (n\psi + 1)(n + 1)(n - \psi + 1)(m + 2)^2 - \varphi m + 1)(n + 1)(n - \psi + 1)(m + 2)^2 - (1 - \varphi)m((n\psi + 1)(n + 1)(m + 2)^2 + (\varphi m + 1)(m + 1)(n + 2)^2 - (1 - \varphi)m(n\psi + 1)(n + 1)(m + 2)^2 + (n - n\psi)(m + 1)(n + 2)^2 - (n\psi + 1)(n + 1)(1 + \varphi m)) - (n - n\psi)(m + 1)(n + 2)^2(1 + \varphi m))^2}{(n + 2)[b\delta(m + 2)^2 - n(1 - \psi)(m + 1)(2 + \varphi m)] - (n\psi + 1)(n + 1)(n - \psi + 1)(m + 2)^2 - (1 - \varphi)m(n\psi + 1)(n + 1)(m + 2)^2 + (n - n\psi)(m + 1)(n + 2)^2}.
\]

\[
x^*_s = \frac{(a - \alpha - 2\tau)[(n\psi(n + 1)(m + 2) + (m + 1)(n + 2)^2]((a - \alpha - 2\tau)\varphi m + a - \alpha + m\tau) - (n\psi + 1)(n + 1)(n - \psi + 1)(m + 2)^2 - \varphi m + 1)(n + 1)(n - \psi + 1)(m + 2)^2 - (1 - \varphi)m((n\psi + 1)(n + 1)(m + 2)^2 + (\varphi m + 1)(m + 1)(n + 2)^2 - (1 - \varphi)(1 - \psi)mn((n + 1)(m + 2)^2(1 + n\psi) + (m + 1)(n + 2)^2(1 + \varphi m))^2}{(n + 2)[b\delta(m + 2)^2 - n(1 - \psi)(m + 1)(2 + \varphi m)] - (n\psi + 1)(n + 1)(n - \psi + 1)(m + 2)^2 - (1 - \varphi)m(n\psi + 1)(n + 1)(m + 2)^2 + (n - n\psi)(m + 1)(n + 2)^2 - (n\psi + 1)(n + 1)(1 + \varphi m)) - (n - n\psi)(m + 1)(n + 2)^2(1 + \varphi m))^2}{(n + 2)[b\delta(m + 2)^2 - n(1 - \psi)(m + 1)(2 + \varphi m)] - (n\psi + 1)(n + 1)(n - \psi + 1)(m + 2)^2 - (1 - \varphi)m(n\psi + 1)(n + 1)(m + 2)^2 + (n - n\psi)(m + 1)(n + 2)^2 - (n\psi + 1)(n + 1)(1 + \varphi m)) - (n - n\psi)(m + 1)(n + 2)^2(1 + \varphi m))^2}.
\]

Taking a derivative of \( x^*_h \) and \( x^*_s \) with respect to each of the parameters \( m, n, \varphi \) and \( \psi \), we obtain a ratio, where the denominator is unambiguously positive while the sign of the numerator is determined by the sign of a cubic polynomial in \( \delta \). As soon as \( \delta \) is sufficiently large – greater than the largest
real root of the polynomial, the sign of the polynomial is defined by the sign of the coefficient at the highest degree.

Thus, to simplify calculations, I assume that \( \delta \) is large enough \( (\delta > \Delta) \) and focus on the sign of the polynomial’s coefficient at \( \delta^3 \). I obtain that under the parameter restriction (9.6), partial derivatives \( \frac{\partial s^*}{\partial m} \), \( \frac{\partial r^*}{\partial m} \), and \( \frac{\partial t^*}{\partial m} \) are negative and the derivative \( \frac{\partial s^*}{\partial \varphi} \) is positive. As regarding the derivative \( \frac{\partial s^*}{\partial \varphi} \), this derivative is positive if and only if the following inequality holds:

\[
(a - \alpha - 2\tau)(1 - \varphi) \cdot A + (a - \alpha - 2\tau) \cdot B + \tau \cdot C > (a - \alpha - 2\tau)(1 - \varphi) \cdot D - (a - \alpha - 2\tau) \varphi \cdot E,
\]

where

\[
\begin{align*}
A &= m^6n^4 + 7m^6n^3 + 18m^6n^2 + 20m^6n + 8m^6 + 12m^5n^4 + 84m^5n^3 + 216m^5n^2 + 240m^5n + 96m^5, \\
B &= -30m^4n^4 \varphi + 50m^4n^3 \varphi + 300m^4n^3 \varphi + 840m^4n^2 \varphi + 1000m^4n^2 - 960m^4n \varphi \\
&+ 1120m^4n - 384m^4 \varphi + 448m^4 + 40m^3n^4 \varphi + 100m^3n^4 - 320m^3n^3 \varphi + 880m^3n^3 \\
&- 1280m^3n^2 \varphi + 2400m^3n^2 - 1600m^3n \varphi + 2720m^3n - 640m^3 \varphi + 1088m^3 + 240m^2n^4 \varphi \\
&+ 120m^2n^4 + 240m^2n^3 \varphi + 1200m^2n^3 - 480m^2n^2 \varphi + 3360m^2n^2 - 960m^2n \varphi + 3840m^2n \\
&- 384m^2 \varphi + 1536m^2 + 288mn^4 \varphi + 112mn^4 + 576mn^3 \varphi + 1024mn^3 + 384mn^2 \varphi + 2816mn^2 \\
&+ 3200mn + 1280m + 16n^5 \varphi + 96n^4 \varphi + 64n^4 + 192n^3 \varphi + 448n^3 + 128n^2 \varphi + 1152n^2 + 1280n + 512, \\
C &= 160m^4 + 1024m + 1280n + 768m^2 + 256m^3 + 32m^4 + 1280n^2 + 640n^3 + 512 + 16n^5 \\
&+ 1929m^2n^2 + 960m^2n^3 + 640m^3n^2 + 320m^3n^3 + 80m^4n^2 + 24m^2n^5 + 80m^3n^4 + 40m^4n^3 \\
&+ 1286mn^3 + 8m^3n^5 + 10m^4n^4 + m^4n^5 + 2560mn + 2560mn^2 + 1920m^2n + 640m^3n \\
&+ 80m^4n + 32mn^5 + 32mn^4 + 240m^2n^4, \\
D &= m^4n^5 + 6m^3n^5 + 12m^2n^5 + 8mn^5, \\
E &= 2m^4n^5 + 14m^3n^5 + 36m^2n^5 + 40mn^5.
\end{align*}
\]

Notice that \( A, B, C, D, \) and \( E \) are all positive, so that the left-hand side of (9.8) is positive, while the sign of the right-hand side is determined by relative values of \( (1 - \varphi) \cdot D \) and \( \varphi \cdot E \). It is easy to see that \( 2D < E \). Hence, for \( \varphi \geq 1/3 \), \( (1 - \varphi) \cdot D < \varphi \cdot E \) and the right-hand side of (9.8) is negative. This establishes condition (b) of the proposition.

Observe also that \( C > D \). Then as soon as \( \tau \geq (a - \alpha - 2\tau)(1 - \varphi) \), inequality (9.8) holds. This justifies condition (a).

Finally, condition (c) follows from the series of inequalities. First, when \( n \leq m^2 \),

\[
A > m^4n^5 + 12m^3n^5 + 7m^2n^5 + 84mn^5.
\]

(9.9)
Secondly, since \( m > n \),

\[
m^4 n^5 + 12m^3 n^5 + 7m^2 n^5 + 84mn^5 > m^4 n^5 + 6m^3 n^5 + 13m^2 n^5 + 84mn^5 > D. \tag{9.10}
\]

Combining (9.9) and (9.10), we obtain that \( A > D \), so that inequality (9.8) is satisfied.

\[\square\]

**Proof of Proposition 3**

First, notice that a complete network of degree \( n \) (\( m \)) can be regarded as a hub-and-spoke network "composed only of hubs", that is, where \( \psi = 1 \) (composed only of spokes where \( \varphi = 1 \)). Then inequality \( x^*_h > x^*(n) \) follows from part 1 of Proposition 2, stating that \( x^*_h \) is decreasing in \( \psi \). Similarly, \( x^*(m) > x^*_s \) is implied by the result that \( x^*_s \) is increasing in \( \varphi \). Lastly, inequality \( x^*(n) > x^*(m) \) follows from Proposition 1.

\[\square\]

**Proof of Proposition 4**

Consider the first series of inequalities in Proposition 4:

\[
x^*_h1 > x^*_h3 > x^*_h4 > x^*(n) > x^*(m) > x^*_s5 > x^*_s3 > x^*_s1.
\]

There, the first three inequalities follow from part 1 of Proposition 2: \( x^*_h1 > x^*_h3 \) since \( x^*_h \) is decreasing in \( m \), while \( x^*_h3 > x^*_h4 \) since \( x^*_h \) is decreasing in \( \psi \). Similarly, the last three inequalities are implied by part 2 of Proposition 2: \( x^*(m) > x^*_s5 > x^*_s3 \) since \( x^*_s \) is increasing in \( \varphi \), while \( x^*_s3 > x^*_s1 \) since \( x^*_s \) is increasing in \( m \). The intermediate inequality \( x^*(n) > x^*(m) \) is a result of Proposition 1.

Likewise, with regard to the equilibrium R&D efforts \( x^*_h2 \) and \( x^*_s2 \) in Type 2 system, the inequality \( x^*_h1 > x^*_h2 \) follows from the fact that \( x^*_h \) is decreasing in \( \psi \), while \( x^*_h2 > x^*_h4 \) and \( x^*_s4 > x^*_s2 \) are the result of \( x^*_h \) and \( x^*_s \) being decreasing and increasing in \( m \), respectively.

\[\square\]

**Proof of Proposition 5**

The proof is suggested by the proof of Lemma 2.3 and Lemma 7.1 in Bloch and Quérou (2008).

Consider the system of linear equations (9.1). Since \( \|\lambda B\| \leq \frac{\xi}{N} < \frac{1}{N} \), Lemma 7.1 in Bloch and Quérou (2008) states that (9.1) possesses a unique solution and

\[
\|x^* - \lambda \tilde{u} \cdot \sum_{k=0}^{K} \lambda^k B^k\| \leq \frac{\lambda N^{K+1} \|\lambda B\|^{K+1} \|\tilde{u}\|}{1 - N \|\lambda B\|} \leq \frac{\lambda \xi^{K+1} \|\tilde{u}\|}{1 - \xi}.
\]

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Observe that \( c^m \) is defined so that

\[
\lambda \hat{u} \sum_{k=0}^{K} \lambda^k B^k = \sum_{m=1}^{K+1} c^m.
\]

So,

\[
\|x^* - \sum_{m=1}^{K+1} c^m \| \leq \frac{\lambda \varepsilon K+1 \| \hat{u} \|}{1 - \varepsilon}.
\]

By definition of the \( l_\infty \) vector norm, this means that \( \forall i \in 1: N \)

\[
|x^*_i - \sum_{m=1}^{K+1} c^m_i| \leq \frac{\lambda \varepsilon K+1 \| \hat{u} \|}{1 - \varepsilon}.
\]

(9.11)

Consider a pair \((i,j)\) of players and let \( M \) be the first element of the sequences \( f_i, f_j \) such that \( c^M_i \neq c^M_j \). Applying (9.11) to \( i \) and \( j \), we obtain

\[
|x^*_i - x^*_j - (c^M_i - c^M_j)| \leq 2 \cdot \frac{\lambda \varepsilon K+1 \| \hat{u} \|}{1 - \varepsilon}.
\]

This concludes the proof.

Appendix C: Figures

![Figure 5: Equilibrium R&D efforts in the hub-and-spoke trade system as a function of \( n \) (the upper sub-figure) and as a function of \( m \) (the lower sub-figure).](image-url)
Figure 6: Equilibrium R&D efforts in the hub-and-spoke trade system as a function of $\psi$ (the upper sub-figure) and as a function of $\phi$ (the lower sub-figure).

Figure 7: Equilibrium R&D efforts in Type 2 system as compared to R&D efforts in other hub-and-spoke systems and to R&D of a country in the multilateral agreement.
Figure 8: Aggregate equilibrium R&D efforts of $n$ countries in the star and in the multilateral agreement.

References


